The Calculus Conundrum

So there I was…

I was talking to another calculus teacher, Mrs. AP Calca, and she relayed this story to me.

Gather around and I will tell you a tale about the horrendous events of this past weekend. You may notice my bedraggled appearance accented by the dark circles under my eyes. I haven’t slept a wink in two days. Someone committed some dastardly deeds which scared me half to death. I am sorry to say that the culprit may be someone you have heard of before. Read the account which follows and see whether you can determine the identity of the perpetrator from the following list of suspects. Cross of suspects as you go. Whomever is not crossed off is the guilty suspect. Or said another way… whomever you derive to be guilty will become the prime suspect… (get it?)

1. Georg Riemann
2. Michel Rolle
3. I Sac Newton
4. Pierre de Fermat
5. Marquis de L’Hospital
6. Wilhelm Leibniz
7. Rene Descartes
8. Aria Betweencurves
9. Derry Vative
10. Int E Gruel
On Friday afternoon, I left the building quite late after working on a set of calculus tests. As I walked to my car, I passed by the building and narrowly escaped being struck in the head by a falling object. Sadly, that object was the bust of Sir Isaac Newton, a cherished gift from my calculus class. As I picked up Sir Isaac’s remains, I discovered a note glued to one of the shards. It read:

Mrs. AP Calca:

Lucky for you, it was only Newton’s bust. I’ve had just about enough of your calculus problems. Now I want you to solve some of mine. We’re going to play a little game. Follow orders exactly or you may end up in worse shape than Isaac. Here’s the first question:

Old Isaac took a little nosedive from his perch on top of the building, 25 feet above ground. Given that he fell as a result of a gentle tap to his noggin, how fast is Isaac traveling when he hits the ground? (In case you forgot, since the acceleration due to gravity is -32 ft/sec²: \( s(t) = 0.5 (-32) t^2 + 25 \) and \( s(t) = 0 \) when it hits the ground. Make sure your answer is in feet/sec. Leave your reply under the brick beside your car.

Help me determine the identity of the perpetrator. Solve the previous problem in this space. To eliminate a suspect, divide your answer to the problem by -10 and cross off the name that corresponds to that number on the list.

Suspect # ______

I was afraid not to solve the problem. I pulled a piece of paper from my bag, hurriedly scratched off a solution and located the brick. I hopped into my car and squealed out of the parking lot.
When I woke up the next morning, I decided the previous day's episode must have been a silly prank. So I returned to school to plan Monday's lesson. Of course, since it was Saturday, the building was locked so I had to work in a trailer that was sitting outside. It was an extremely hot day for late April, but the trailer was nice and cool inside. At least, it was cool in the trailer at first. But as I worked, I realized that the air conditioning must have shut down because it just got hotter and hotter by the minute. I tolerated it for about an hour and then, unable to stand it any longer, I packed up my things to leave. I tried to open the door of the trailer but it was jammed. Then a note appeared under the door. Here’s how it read:

Getting a mite warm, isn’t it? I’ve been keeping track of the temperature. Here’s a table of the readings that I’ve taken of the temp in the trailer starting at 10:00 a.m. when I shut off the AC.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (F)</td>
<td>68</td>
<td>72.64</td>
<td>74.97</td>
<td>76.31</td>
<td>82.9</td>
<td>85.38</td>
<td>87.5</td>
</tr>
</tbody>
</table>

Use a Midpoint approximation for the average temperature in the trailer from 10:00 until 11:00. Remember to use the average value of a function formula. Slide your answer under the door or you’ll be sweating for a long time. You have 2 minutes to solve this!

Eliminate another suspect: It’s the digit that appears in your answer twice.

Suspect # ______
I solved the problem as quickly as I could and slid it under the door. But I guess it wasn’t fast enough. Here’s the note I got back:

That was 3 minutes, Mrs. AP. You blew it, but here’s another chance. . . .

At that point I started to panic. I thought to myself, “Maybe I can escape through the window.” But the window turned out to be jammed too, and I had nothing to break it with but my bare fist. Finally, the trailer door clicked open. I looked out the window and saw a lone figure running off through the trees. I dropped everything, ran out the door, and made a beeline for my car. As I climbed into the driver’s seat, the doors automatically locked! There on the steering wheel was . . . I bet you can guess . . . another note! I turned the key and . . . nothing.

I took the hood ornament from your car and replaced it with one I made myself. Your old one was so boring - like you! Do you recognize it? It’s a solid with a base formed by intersecting sine and cosine curves (between their first two points of intersection in terms of π) that have positive x-values) and built up with square cross sections perpendicular to the x-axis. I want you to find two things:

1. The area of the base
2. The volume of the solid

When you’re done, add your answers and beep your horn that many times. If you are right, I will release the mechanism that has disabled your engine.

Eliminate two more suspects. Find the answers to 3 decimal places and then do the following:

For question #1, take the first digit of your answer.

For question #2, double the digit in the hundredths place.
I worked furiously because I just wanted to get out of there. When I got my answers, I beeped the horn and a big white flag unfurled from a tree across the street. Here’s what it said:

Within seconds, the motor of my car cranked on. I gunned the engine and sped off. Much to my dismay, the radio came on and I heard these words:

Congratulations! You need a bigger challenge. Since you can’t write and drive at the same time you have to do this one in your head. Look out the window of your car. I replaced the orange ping pong ball on your antenna. You needed a new way to find your car in Walmart’s parking lot. I carved a parabola out of the cover of your teacher’s edition. Notice how nicely it spins around the antenna in the wind, you old windbag.

Find the volume of this solid of revolution if the equation of the parabola is \( x = y^2 \) and the antenna is the y-axis. Write your answer on the windshield with lipstick. It better be up there by the time you cross the Yough or you may just go for a swim.

Eliminate another suspect. You must find an exact answer. The denominator of your answer is the suspect to cross off….

This was easy for me because I had worked that very problem just the night before, and I had the answer memorized. I fumbled through my purse for lipstick. Reluctantly, I scribbled the answer on the windshield. I crossed the Yough and then I heard a siren and saw flashing lights in my rear view mirror. The cop pulled me over and said, “Lady, are you drunk?!? Why are you writing numbers on your windshield in lipstick?” I told him I was being tormented by a lunatic calculus student who locked me in a trailer, broke my Isaac Newton, and stole my ping pong ball. He said, “Step out of your vehicle and walk this line.” After a humiliating couple of minutes walking backward and forward on the side of the road, the cop let me go with this observation, “You need help lady.”
Finally, I arrived home and pulled into my driveway. Exhausted, I dragged myself up to the front door and found a note hanging on the knob. Here’s what it said:

While you were walking the line for the cop, I was hiding in the bushes with my radar gun. I made a graph of your velocity in feet per minute during your little 3-minute jaunt. At time = 0, you were backed up against the bumper of your car. Answer these questions (correct to 3 decimal places) or your pet canary has sung its last note.

1. How far were you from your car 2 minutes after you started walking?
2. What was your acceleration 1 minute into your walk?
3. What was your average velocity over the first minute?

Eliminate another suspect: Find each answer. Then find the sum of the three answers. Take the first digit of that sum and cross off that suspect.

Suspect # ______
I went inside and securely bolted the door. I was relieved to find nothing out of place and my precious canary, Archimedes, unharmed. I went into the kitchen for a cup of tea to soothe my nerves. I gulped it down. As I sat down at the kitchen table, I saw, for the first time, two packages. One was beautifully wrapped and tied with a shiny pink ribbon shaped like this:

![Image of a beautifully wrapped package](image)

The other was shabbily constructed and covered with aluminum foil. I was afraid to open either one. Then I saw the writing on the ribbon.

![Image of a shabbily constructed package](image)

“This is ridiculous,” I thought. I started to reach for the other package when I heard a resounding boom at my kitchen window. A large wad of putty was smooshed against the glass. Hanging from the wad was a sign that looked like this:

![Image of a sign saying DO FOLIUM NOW!](image)

Eliminate another suspect: Find the exact value of the problem. Square the answer and cross off the suspect whose number is the same as the denominator.
So I worked the problem. I didn’t know what would happen to me if I didn’t do it. I taped my answer to the window and waited. Nothing happened, so I figured it was safe to open the other box. Under the aluminum foil, I found this note:

I filled this box with your favorite cookies - fig NEWTONs. Your pal Isaac sure was a busy little inventor! Of course, I made the box myself by cutting equal squares from the corners of a 10 by 14 inch rectangular piece of manilla folder which I stole from your classroom. Using the quadratic formula in my solution I cut out the corners (rounded to the nearest whole number of inches) so as to maximize the number of cookies I could fit into the box. Given that each fig Newton is 2 inches by 1 inch by 0.5 inch, how many fig Newtons could I fit in the box? (Assume that each cookie must be fully intact.)

Eliminate another suspect: To the nearest dozen, how many cookies are in the box?

I was sick of this game. I knew the police would never believe me, and out of desperation I was gaining courage. I decided to take matters into my own hands. I turned out the lights and went down to the basement. I opened the door a wee bit and peered around the yard. No one was in sight but I knew the punk was nearby. I exited quickly, my flashlight in the waistband of my pants. I could make out the outline of a body up against the concrete wall in the back corner of the yard. I knew all about estimating rates and distances.
I knew if the beam of a flashlight were aimed at a wall, it would form a cone whose altitude would change at the same rate that I approached the wall. If the volume of the cone of light was decreasing at a rate of 16 cubic feet per second, I figured the radius of the beam would increase at a rate of 7 ft/sec and the height decrease at 6 ft/sec. I asked myself, “How far (to the nearest foot) from the scoundrel would I have to be for all of these facts to come together and illuminate him (or her??) in an 8 foot radius of light?” The distance I would be away from the suspect when illuminated would be the height of the cone. Volume of a cone = $\frac{1}{3}\pi r^2 h$

Use your answer to eliminate another suspect.

I got within 5 feet and – to my dismay – I had miscalculated. My tormentor sprang up and ran off into the night. And that was the end of my ordeal.

I have related this tale to you, hoping that you might help me discover the identity of my assailant. If you followed my tale and possess the mathematical skills to answer all the problems correctly, you will now know the identity of the culprit and be a true calculus nerd!

Who Dun It?  ________________________________